Macroeconomic theory, Module 3
Problem Set 1

The due date for this problem set is Wednesday 31.1.2018 BEFORE the exercise session. Not accepted later.

1. Embedding population growth in the stochastic growth model

Assume that the labor force grows at some exogenous gross rate (γ_L)

$$\begin{aligned} \tilde{L}_t &= \gamma_L^t \tilde{L}_0, \ \tilde{L}_0 = 1 \\ &\Rightarrow \ \tilde{L}_t = \gamma_L^t \end{aligned}$$

and that output is given by

$$\tilde{Y}_t = A_t \tilde{K}_t^{\alpha} (\tilde{X}_t \tilde{L}_t)^{1-\alpha}$$

where $\tilde{X}_t$ is labor-augmenting technology. Further assume that $\tilde{X}_t$ grows at a constant, exogenous gross rate so that

$$\begin{aligned} \tilde{X}_t &= \gamma_X^t \tilde{X}_0, \ \tilde{X}_0 = 1 \\ &\Rightarrow \ X_t = \gamma_X^t \end{aligned}$$

If the TFP shock process A_t is shut down, the model converges towards a balanced growth path, where the capital stock, output, consumption and investment all grow at a common, constant rate, given by$\gamma_L \gamma_X$.
When we analyze fluctuations around the balanced growth path, the basic idea is to reformulate the optimization problem terms of variables that are stationary (and constant on the non-stochastic balanced growth path). Detrending simply involves dividing the variables by the growth trend. In particular

$$Y_t = \frac{\tilde{Y}_t}{\gamma_L^t \gamma_X^t}, C_t = \frac{\tilde{C}_t}{\gamma_L^t \gamma_X^t}, K_t = \frac{\tilde{K}_t}{\gamma_L^t \gamma_X^t},$$

are then detrended output, consumption and capital (in period t).

a) The resource constraint of the model with trend growth is

$$\tilde{K}_{t+1} - (1-\delta)\tilde{K}_t + \tilde{C}_t = \tilde{Y}_t$$

Re-write the resource constraint in terms of the detrended variables.
b) Assume that the representative household, or the social planner, maximizes the following intertemporal objective function

$$E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \gamma_L^t U(\hat{C}_t)$$

where $\hat{C}_t = \frac{\tilde{C}_t}{\gamma_L^t}$ is *per capita* consumption. (Notice that the size of the representative household grows at gross rate γ_L.) Further assume that the utility function is CRRA $U(\hat{C}_t) = \frac{\hat{C}_t^{1-\sigma}}{1-\sigma}$. Show that the objective function can be re-expressed with *detrended* consumption C_t

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}$$

Express the modified discount factor β in terms of $\tilde{\beta}$, σ, γ_L and γ_X.

c) Write down the social planner's Bellman equation problem (without solving it) in terms of the detrended variables. What are the exogenous and endogenous state variables ?

d) Cast the planner's optimization problem in the Lagrange multiplier form and solve it. In particular, derive the intertemporal Euler equation. What is the long-run (or balanced growth path) interest rate of this economy? Does the long-run interest rate depend on population growth (γ_L) and/or on the growth rate of (labor-augmenting) technology γ_X?

e) Log-linearize the model around the non-stochastic steady state

f) Compute the "great ratios" $\left(\frac{\bar{K}}{\bar{Y}}\right)$, $\left(\frac{\bar{C}}{\bar{Y}}\right)$ and $\left(\frac{\bar{I}}{\bar{Y}}\right)$ along the balanced growth path. Do these great ratios depend on population growth γ_L and/or on the growth rate of (labor-augmenting) technology γ_X?

2. Redo the calculations reported in "Table 1: Business Cycle Statistics for the US economy" in the Lecture 1-2 notes for an other country than the US. You can choose almost any country that you want, except that the first letter of the country you choose has to be contained in your last name (e.g. for Jones admissible countries start with letters E,J,N,O and S). You should report results for the series Y, C, I, N, $\frac{Y}{N}$ and A (ignore w and r). Data on consumption may be total consumption (including durables). Compute all 4 moments reported in the table. Use Matlab whenever possible. Please document carefully every step you make and submit your Matlab code (think of somebody who would want to reproduce your results from scratch). Plot the series, original and HP-filtered. Comment on your results. Are they similar to the "stylized facts" for the U.S.?

The data should be logged (natural logarithms) and then HP-filtered. Work with MOST UP-TO-DATE QUARTERLY data. Obtain as longest series as possible for your country. Make sure your data is real and seasonally adjusted. If necessary, seasonal adjustment may be done in Eviews. If you cannot obtain seasonally adjusted data or cannot use Eviews, use the BK filter instead of HP. If you can only get annual data (this may be the case for hours worked), interpolate annual values into quarterly values using your preferred interpolation method. The Matlab HP-filter function should be readily available in Matlab but it can also be easily found online.

The more complicated part will involve obtaining the series for capital (necessary to compute TFP). The data on capital stock is usually incomplete, at only annual frequency, shorter than the series for I or non-existent at all. To construct capital stock series at quarterly frequency on can use then the "perpetual inventory method". If you have data for K in a particular year (or quarter) matching the beginning or end of your I sample, you can use the capital motion equation

$$K_{t+1} = (1-\delta) K_t + I_t$$

combined with your investment data to recover the quarterly capital stock series. Here, one has to assume some depreciation rate. A common number is $\delta = 0.025$ quarterly (or 10% annual depreciation). If you cannot get any data on the capital stock to use as a starting point, you may assume that in the first quarter of your available investment series the economy was in the steady state. The corresponding value of capital was then $K_0 = \frac{I_0}{\delta}$.

Given the series for hours and capital one can obtain the growth rate of the Solow residual (proxy for A) by regressing the growth rate of real GDP on the growth rates of capital stock and total hours worked. The trending level of TFP (which will have to be later HP-filtered) can then be recovered from these obtained TFP growth rates.

Commonly used, freely available, sources macro data include, among others,: IMF eLibrary, Eurostat, OECD, St. Louis FED FRED, ILO (for labor data), IDB Latin Macro Watch (for Latin America), national statistics offices and central banks.