Macroeconomic theory, Module 3
Problem Set 2

The due date for this problem set is Wednesday 13.2.2019 at the beginning of the exercise session. Not accepted later.

Trend shocks

Consider a simple stochastic growth model in which the representative agent maximizes the following utility function:

$$\max_{\{C_t,L_t,I_t,K_{t+1},B_{t+1}\}} E\sum_{t=0}^{\infty}\beta^t \frac{\left[C_t^{\theta}\left(1-L_t\right)^{1-\theta}\right]^{1-\sigma}}{1-\sigma}$$

subject to the budget constraint.

$$C_t + B_{t+1} = R_t B_t + \Pi_t + W_t L_t \tag{1}$$

All variables have their usual meaning. Labor supply is fixed:

$$L_t = \bar{L} \tag{2}$$

The production technology is given by

$$Y_t = A_t K_t^{\alpha}\left(\Gamma_t L_t\right)^{1-\alpha} \tag{3}$$

where

$$\ln A_t = (1-\rho_A)\ln A + \rho_A \ln A_{t-1} + \epsilon_t^A \tag{4}$$

is the standard stationary TFP shock and where the law of motion for capital is

$$K_{t+1} = (1-\delta)K_t + I_t \tag{5}$$

The novel part of the model is that the non-stationary trend Γ_t is no longer deterministic, but stochastic. In particular, the (gross) growth rate of the trend γ_t may be different in every period and is subject to shocks:

$$\frac{\Gamma_t}{\Gamma_{t-1}} = \gamma_t; \quad \Gamma_0 = 1; \quad \text{so} \quad \Gamma_t = \prod_{s=1}^{t}\gamma_s \tag{6}$$

$$\ln\gamma_t = (1-\rho_\gamma)\ln\gamma + \rho_\gamma\ln\gamma_{t-1} + \epsilon_t^{\gamma} \tag{7}$$

where γ is the mean of γ_t.

a) Detrend all relevant variables using the *previous*-period trend level and denote them with their lower-case counterparts:

$$x_t = \frac{X_t}{\Gamma_{t-1}} \tag{8}$$

Remember about the proper adjustment of the subjective discount factor β.

b) Specify the Bellman problem of the planner in terms of the detrended variables. Find the first-order and envelope conditions of the Bellman problem.

c) Specify the two maximization problems of households and firms using the Lagrangian. Solve both infinite-horizon problems. Does the Bellman-based solution of the planner coincide with the Lagrangian solution for the market economy? Why or why not? Write down the definition of the equilibrium of this economy.

d) Solve for the NSSS. How many restrictions do you need to impose in the process of calibration to obtain a unique NSSS solution? What restrictions do you choose to impose? Justify your choice.

e) (Log-)linearize the detrended model.

f) Discuss in words what may be the reason for introducing the non-stationary trend shock into this model? How would the the reaction of the economy differ following a positive shock to A_t versus to γ_t processes? Plot a qualitative picture of the trend which experiences a one-time shock somewhere in the middle of the sample (so $\epsilon_t^\gamma \neq 0$ for some t and zero otherwise). How would this graph change if the persistence of the trend shock went up (say, from $\rho_\gamma = 0.8$ to $\rho_\gamma = 0.95$)?

g) cast the linear version of the model into the "brute-force" form of Uhlig's solution method:

$$0 = E_t \left[F x_{t+1} + G x_t + H x_{t-1} + L z_{t+1} + M z_t \right] \tag{9}$$

$$z_{t+1} = N z_t + \epsilon_{t+1}; \quad E_t \left[\epsilon_{t+1} \right] = 0 \tag{10}$$

i.e. specify the coefficient matrices F, G, H, L, M and N. x_t contains all "jumping" variables, i.e. controls chosen at t. x_{t-1} contains predetermined endogenous states known before time t shocks are realized, and $z_t + 1$ contains all exogenous states the value of which is realized in $t + 1$, i.e. when the $t + 1$ shock hits. By analogy, z_t are exogenous states the value of which is realized in period t.