Macroeconomic theory, Module 3
Problem Set 2

The due date for this problem set is Wednesday 7.02.2018 BEFORE the exercise session. Not accepted later.

1. RBC model with fixed labor supply

Consider the following RBC model with fixed labor supply:

$$\max_{\{C_t, L_t, K_{t+1}, I_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\theta \ln C_t + (1-\theta)\ln(1-L_t)\right]$$

$$\begin{aligned}
& s.t. \\
C_t + I_t &= A_t K_t^{\alpha} L_t^{1-\alpha} \\
K_{t+1} &= I_t + (1-\delta)K_t - \frac{\psi}{2}\left(\frac{K_{t+1}}{K_t} - 1\right)^2 K_t \\
\ln A_t &= \rho \ln A_{t-1} + (1-\rho)\ln \bar{A} + \epsilon_t, \;\; \epsilon_t \sim N(0, \sigma_\epsilon^2) \\
L_t &= \bar{L}, \;\; \forall\, t
\end{aligned}$$

with the usual notation, $\rho \in (0,1)$.

a) Using L'Hopital's rule show that the instantaneous utility function $[\theta \ln C_t + (1-\theta)\ln(1-L_t)]$ is a special case of the function $\frac{[C_t{}^{\theta}(1-L_t)^{1-\theta}]^{1-\sigma}}{1-\sigma}$ when $\sigma = 1$.

Hint: L'Hopital's rule postulates that

$$\text{if } \lim_{x\to c} f(x) = \lim_{x\to c} g(x) = 0 \text{ then } \lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$$

b) Suppose the economy is hit by a positive shock to the stationary total productivity process A_t. Without having seen any impulse response functions, discuss your expectations regarding the reaction of the model's variables to the shock. In particular,

- which variable should react more strongly (on impact and in a few periods following the shock), Y_t or A_t? Why?
- which variable should react more strongly (on impact and in a few periods following the shock), consumption C_t or output Y_t? Why?
- which variable should react more strongly (on impact and in a few periods following the shock), investment I_t or output Y_t? Why?
- how much will capital K_t react on impact (in the period in which the shock hits)? Why?

c) Now suppose that the economy is hit by the same shock, but the persistence of the shock ρ is **smaller** than the one in the previous point. How do you expect your impulse responses to change? In particular,

- would the reaction of investment I_t be stronger or weaker on impact, relative to the previous case? Will it die out faster or slower? Why?
- would the reaction of consumption C_t be stronger or weaker on impact, relative to the previous case? Will it die out faster or slower? Why?

d) discuss briefly how would your conclusions from **point b)** change if labor supply was variable instead of fixed?

e) The term $\frac{\psi}{2}\left(\frac{K_{t+1}}{K_t}-1\right)^2 K_t$ denotes capital adjustment cost of the model. What is the usual purpose of introducing such costs to RBC models? How does the cost depend on ψ?

2. Solving the model
Consider again the model from the previous exercise.

a) which of these variables in the model are endogenous states, exogenous states and which are controls?

b) derive the optimality conditions for the choice of consumption, capital and labor

c) Solve for the NSSS. How many restrictions do you need to impose in the process of calibration to obtain a unique NSSS solution? What restrictions do you choose to impose? Justify your choice.

d) (log-) linearize the model around the NSSS.

e) cast the linear version of the model into the "brute-force" form of Uhlig's solution method:

$$0 = E_t\left[Fx_{t+1} + Gx_t + Hx_{t-1} + Lz_{t+1} + Mz_t\right] \tag{1}$$

$$z_{t+1} = Nz_t + \epsilon_{t+1}; \quad E_t\left[\epsilon_{t+1}\right] = 0 \tag{2}$$

i.e. specify the coefficient matrices F, G, H, L, M and N. x_t contains all "jumping" variables, i.e. controls chosen at t. x_{t-1} contains predetermined endogenous states known before time t shocks are realized, and $z_t + 1$ contains all exogenous states the value of which is realized in $t+1$, i.e. when the $t+1$ shock hits. By analogy, z_t are exogenous states the value of which is realized in period t.