Macroeconomic theory
Problem Set 3

The due date for this problem set is Wednesday 20.02.2019 AT THE BEGINNING OF the exercise session. Not accepted later. Alternatively, please send the homework to kristine.koponen@helsinki.fi (the same deadline).

1. RBC model with endogenous labor supply

Consider the following RBC model with endogenous labor supply:

$$\max_{\{\tilde{C}_t, H_t, \tilde{K}_{t+1}, \tilde{I}_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \frac{\left[\tilde{C}_t^{\theta}(1-H_t)^{1-\theta}\right]^{1-\sigma}}{1-\sigma}$$

$$s.t.$$

$$\tilde{C}_t + \tilde{I}_t = A_t \tilde{K}_t^{\alpha} \left(\tilde{X}_t H_t\right)^{1-\alpha} \quad (1)$$

$$\tilde{K}_{t+1} = \tilde{I}_t + (1-\delta)\tilde{K}_t \quad (2)$$

$$\ln A_t = \rho \ln A_{t-1} + (1-\rho)\ln \bar{A} + \epsilon_t, \ \ \epsilon_t \sim N(0, \sigma_{\epsilon}^2)$$

with the usual notation, $\rho \in (0,1)$ and $\sigma > 1$. $\tilde{X}_{t+1} = \gamma_X \tilde{X}_t$ denotes the nonstationary deterministic trend of the economy.

a) Show that the instantaneous utility function $\frac{\left[\tilde{C}^{\theta}(1-H)^{1-\theta}\right]^{1-\sigma}}{1-\sigma}$ can be re-expressed in the form

$$\frac{\left[\tilde{C}\mu(H)\right]^{1-\eta}}{1-\eta}$$

where $1-\eta = \theta(1-\sigma)$. In particular, find the explicit formula for $\mu(H)$.

b) Without detrending the model derive the optimal labor supply as a function of wages. At what rates do all the variables in that function grow along the balanced growth path? How then does the steady state labor supply $\bar{H}$ depend on the level of technology $\tilde{X}$? Recall what is the major long-run stylized fact about labor supply and say if this labor supply function is consistent with this fact?

c) Consider now a detrended version of the model. Suppose the economy is hit by a positive shock to the stationary total productivity process A_t. Without having seen any impulse response functions, discuss your expectations regarding the reaction of the model's variables to the shock. In particular,

- which variable should react more strongly (on impact and in a few periods following the shock), Y_t or A_t? Why?
- which variable should react more strongly (on impact and in a few periods following the shock), consumption C_t or output Y_t? Why?
- which variable should react more strongly (on impact and in a few periods following the shock), investment I_t or output Y_t? Why?
- will labor supply go up or down on impact and in a few periods following the shock? Why?
- how much will capital K_t react on impact (at the time the shock hits)? Why?

d) Now suppose that the economy is hit by the same shock, but the persistence of the shock ρ is higher than the one in the previous point. How do you expect your impulse responses to change? In particular,

- would the reaction of investment I_t be stronger or weaker on impact, relative to the previous case? Will it die out faster or slower? Why?
- would the reaction of consumption C_t be stronger or weaker on impact, relative to the previous case? Will it die out faster or slower? Why?
- would the reaction of labor H_t be stronger or weaker on impact, relative to the previous case? Will it die out faster or slower? Why?

e) discuss briefly how would your conclusions from point c) change if labor supply was fixed instead of varying?

2. Solving the model

Consider again the model from the previous exercise.

a) which of these variables in the model are endogenous states, exogenous states and which are controls?

b) derive the optimality conditions for the choice of consumption, capital and labor

c) Solve for the NSSS. How many restrictions do you need to impose in the process of calibration to obtain a unique NSSS solution? What restrictions do you choose to impose? Justify your choice.

d) (log-) linearize the model around the NSSS.

e) cast the linear version of the model into the "brute-force" form of Uhlig's solution method:

$$0 = E_t \left[F x_{t+1} + G x_t + H x_{t-1} + L z_{t+1} + M z_t \right] \tag{3}$$

$$z_{t+1} = N z_t + \epsilon_{t+1}; \quad E_t \left[\epsilon_{t+1} \right] = 0 \tag{4}$$

i.e. specify the coefficient matrices F, G, H, L, M and N. x_t contains all "jumping" variables, i.e. controls chosen at t. x_{t-1} contains predetermined endogenous states known before time t shocks are realized, and $z_t + 1$ contains all exogenous states the value of which is realized in $t + 1$, i.e. when the $t + 1$ shock hits. By analogy, z_t are exogenous states the value of which is realized in period t.