Macroeconomic theory, Module 3
Problem Set 3

The due date for this problem set is Wednesday 14.2.2018 BEFORE the exercise session. Not accepted later.

1. Small open economy model with debt-elastic interest rate

Consider a simple small open economy model with the following specification:

$$\max_{\{C_t, I_t, K_{t+1}, B_{t+1}\}} E \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}$$

$$st.$$

$$B_{t+1} + C_t + I_t = A_t K_t^{\alpha} L^{1-\alpha} + R_t^* \Psi_t B_t \quad (1)$$

$$K_{t+1} = (1-\delta) K_t + I_t \quad (2)$$

$$\ln R_{t+1}^* = (1-\rho_{R^*}) \ln \bar{R}^* + \rho_{R^*} \ln R_t^* + \epsilon_t^{R^*} \quad (3)$$

$$\ln A_t = (1-\rho_A) \ln \bar{A} + \rho_A \ln A_{t-1} + \epsilon_t^A \quad (4)$$

where K_t, B_t, R_t^* and A_t are capital, labor, net foreign asset position, world interest rate and TFP, respectively. Labor supply is fixed at $L = 1$. There are no capital installation costs in the model nor is there a trend. However, there is a debt-elastic *interest rate premium*

$$\Psi_t = \left\{ \bar{\Psi} - \tilde{\Psi} \left[\exp \left(B_t^A - B^A \right) - 1 \right] \right\} \quad (5)$$

where $\tilde{\Psi}$ is a parameter governing the elasticity of the risk premium. $\bar{\Psi}$ is the steady state level of the country risk premium. B_t^A is the aggregate level of foreign asset position (equal to B_t in equilibrium), the value of which households take as given (obviously, $B^A = B$ as well). We also assume that $B_t < 0$ so that the country is a net foreign debtor. Function (5) means that decreasing the net foreign asset position from its steady state value is costly and increases the country's cost of borrowing (reducing the debt relative to steady state reduces the premium).

a) what is the purpose of introducing the debt-elastic interest rate function in this class of models? What would happen to the model dynamics if $\tilde{\Psi}$ were $= 0$? (You don't need to make any derivations, just explain in words the logic and intuition)

b) which of these variables in the model are endogenous states, exogenous states and which are controls?

c) derive the optimality conditions for the choice of consumption, capital and net foreign asset position

d) Solve for the NSSS. Assume that the steady state risk premium is strictly > 1. How many restrictions do you need to impose in the process of calibration to obtain a unique NSSS solution? What restrictions do you choose to impose? Justify your choice.

e) (log-) linearize the model under the assumption that the NSSS value of net foreign asset position is strictly negative (i.e. $B = B^A < 0$) and all other variables are strictly positive in NSSS as well.

f) cast the linear version of the model into the "brute-force" form of Uhlig's solution method:

$$0 = E_t \left[F x_{t+1} + G x_t + H x_{t-1} + L z_{t+1} + M z_t \right] \tag{6}$$

$$z_{t+1} = N z_t + \epsilon_{t+1}; \quad E_t \left[\epsilon_{t+1} \right] = 0 \tag{7}$$

i.e. specify the coefficient matrices F, G, H, L, M and N. x_t contains all "jumping" variables, i.e. controls chosen at t. x_{t-1} contains predetermined endogenous states known before time t shocks are realized, and $z_t + 1$ contains all exogenous states the value of which is realized in $t+1$, i.e. when the $t+1$ shock hits. By analogy, z_t are exogenous states the value of which is realized in period t.

2. Computer project on "closing the small open economy".

Consider the model from exercise 1. Suppose that the economy is no longer closed with the debt-elastic risk premium from eq. 5 (i.e. $\Psi_t = 1 \, \forall t$). Instead, there are quadratic costs of changing the level of the net foreign asset position relative to the NSSS, where ψ is a parameter. Hence the budget constraint looks as follows:

$$C_t + I_t + B_{t+1} + \frac{\psi}{2} \left(B_{t+1} - B \right)^2 = A_t K_t^{\alpha} L_t^{1-\alpha} + R_t^* B_t \tag{8}$$

Solve both versions of the model (the one from exercise 1 and the one with convex adjustment costs), find the NSSS, calibrate, log-linearize and solve on the computer.

Report the impulse responses for both models for output, consumption and net foreign asset position following both stochastic shocks. To what extent are the results different? Document carefully your work. Perform stochastic simulations for both versions of the model and discuss the results.

For the software, you may use the Uhlig toolkit (available here):
`https://www.wiwi.hu-berlin.de/de/professuren/vwl/wipo/research/MATLAB_Toolkit`

If you prefer, you can also do it in Dynare, which allows you to skip the linearization step.