RBC model

Macroeconomic Theory, Lectures 5-6

Adam Gulan

Bank of Finland

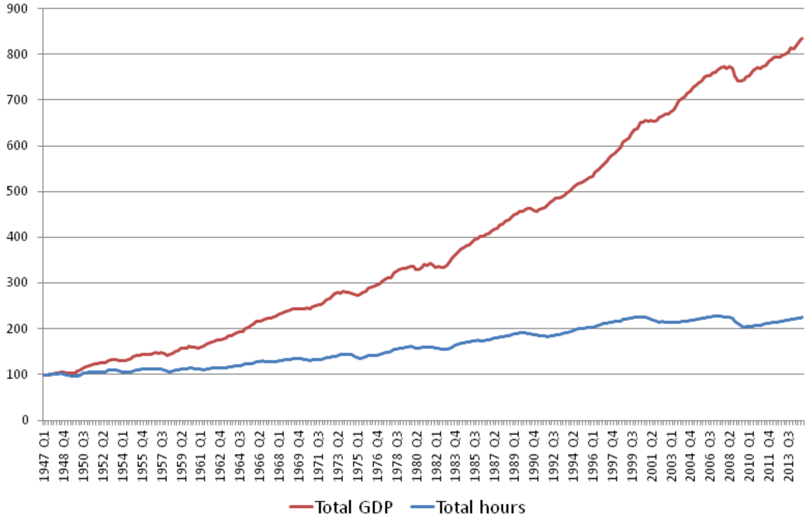
January 2018

Course material

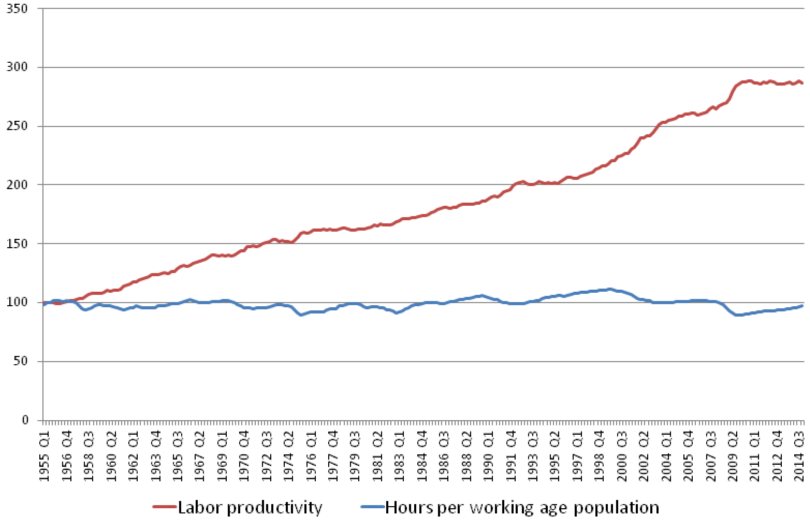
Readings for lectures 5-6:

- McCandless (2008), Ch.4.1, Ch.6
- Romer (2012), Ch.5
- Wickens (2011), Ch.16 (until Ch.16.4.2)
- King, Plosser, Rebelo (JME, 1988), "Production, Growth, and Business Cycles. I. The Basic Neoclassical Model"
- King, Plosser, Rebelo (Computational Economics, 2002), "Production, Growth, and Business Cycles. Technical Appendix"
- DeJong and Dave (2011), Ch.11

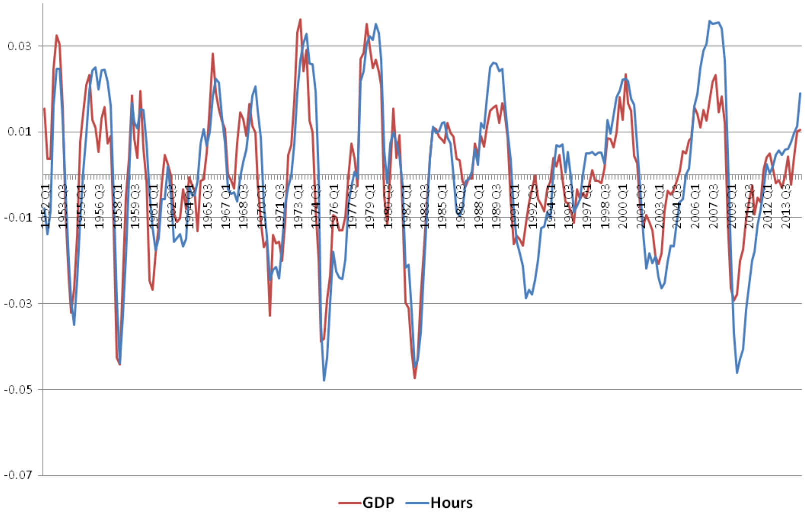
GDP and Hours, USA



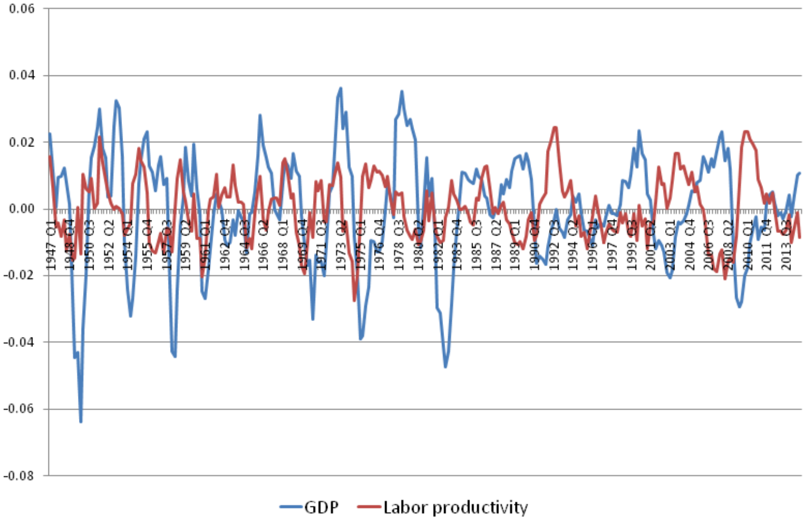
Labor productivity and hours per working age person, USA



GDP and hours, HP cycle, USA



GDP and labor productivity, HP cycle, USA



Output and hours worked

Long run:

- output and labor productivity have grown considerably over time
- but hours worked per capita have remained roughly unchanged over the past decades

Business cycle:

- output and hours worked are tightly knit together

Labor market indicators over the business cycle

Business Cycle Statistics, US (non-farm) (1951Q1-2004Q4)

Variable	**Mean**	**S.E**	**AR(1)**	**Corr(output,x)**
Output	0	2.1%	0.84	1
Total Hours	0	1.8%	0.89	0.87
Average Hours	0	0.7%	0.82	0.72
Employment	0	1.3%	0.92	0.77
TFP	0	1.0%	0.89	0.55
Real wage*	0	0.4%	0.66	0.12

* Sample 1953Q1-1996Q4

Summary of labor market statistics

- TFP fluctuates less than output.
  (We can't generate enough output fluctuations with constant hours, if TFP is main source of BC)
- Total hours fluctuate more than TFP. Intertemporal substitution of labor across periods is an important factor which helps to generate needed fluctuations in total hours.
- Total hours is an important business cycle indicator.
  It is strongly correlated with output.
- A major part of variation in total hours is due to variations in number of employed (extensive margin), as opposed to hours worked (intensive margin).
- Real wages fluctuate considerably less than output and total hours. Real wages are acyclical (correlation with output is close to zero).

Stylized facts and the RBC model

The challenge is to write down a model, in which:

- in the long-run, the increase in productivity (and more generally economic growth) does not affect labor supply
- ...but nevertheless labor supply reacts to short-run fluctuations in productivity (i.e. productivity shocks)

Substitution and income effects

- Higher real wage
  - Substitution effect increases labor supply
  - Income effect decreases labor supply
- If the utility of consumption is logarithmic (unitary elasticity of substitution) so that

$$U\left(C,L\right)=\ln\left(C\right)-v\left(L\right) \tag{1}$$

  where $v\left(L\right)$ is disutility from labor L, the substitution effect and the income effect cancel each other out
- Then steady state labor supply does not depend on real wage rate (and does not depend on labor productivity)

Permanent change in income does not affect labor supply

- Consider a basic maximization problem example:

$$\max_{C,L} \ln(C) - v(L)$$

$$\text{s.t. } C = WL$$

or

$$\max_{L} \ln(WL) - v(L)$$

- The first-order condition is

$$\frac{\overbrace{W}^{\text{substitution effect}}}{\underbrace{WL}_{\text{income effect}}} = v'(L) \Leftrightarrow \frac{1}{L} = v'(L)$$

quite clearly, optimal labor supply L does not depend here on the wage rate W

Utility function more generally

- More generally, for hours worked to be invariant to the level of productivity (or real wage rate), the momentary utility function has to be expressible as

$$U(C, L) = \frac{1}{1-\sigma}\left\{\left[C\mu(L)\right]^{1-\sigma} - 1\right\} \tag{2}$$

  - See King and Rebelo (1999), Resuscitating Real Business Cycles, Section 3.2
  - Notice that setting $\sigma = 1$ and $\mu(L) = e^{v(L)}$ the utility function (2) becomes (1)

- One can also show that on the balanced growth path (where output, capital, consumption and total factor productivity all grow at the same constant rate), labor supply remains unchanged, when the utility function is of the form (2)

Temporary rise in real wage increases labor supply today

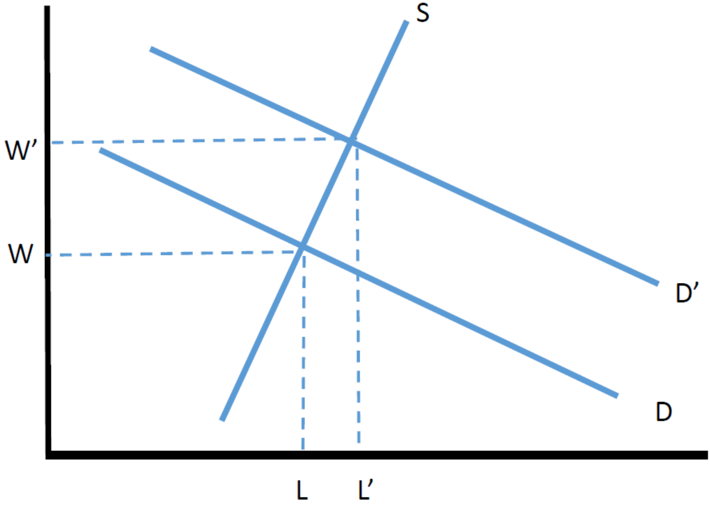
- However, a transitory productivity shock, which temporarily raises the real wage rate, increases labor supply today
  - The substitution effect is strong, while the income effect is very weak
- A transitory increase in real wage rate has almost no effect on *permanent* income (discounted sum of all future revenues)
  - Since individuals are forward-looking the income effect is driven by permanent income, not current income
- Intuitively, people work more today, to be able to consume more tomorrow when productivity (and the wage rate) is expected to be lower
  - A part of today's extra income is saved.
- If instead productivity is expected to be permanently higher, there is no need to work harder today: in any case people can enjoy high consumption in the future (due to the high productivity).

Elasticity of labor supply

- Recall also that the real wage rate varies considerably less over the business cycle than employment.
- Thus (if we want to explain the fluctuations of employment with a model with no labor market frictions) we need a model where the (wage) elasticity of labor supply is high.
- In other words, the elasticity of intertemporal substitution of leisure should be high
  - leisure today and leisure next year are almost perfect substitutes

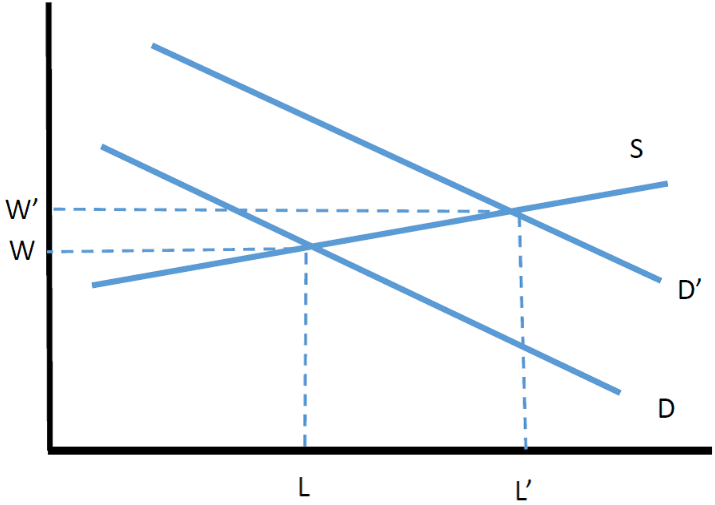
Inelastic labor supply

A positive TFP shock shifts the labor demand curve from D to D':



Elastic labor supply

A positive TFP shock shifts the labor demand curve from D to D':



Elasticity of labor supply, cont.

- However, many empirical studies suggest that hours worked do not react strongly to changes in wage rate
  - Many empirical studies find that the elasticity of labor supply is close to zero.
- These findings are potentially problematic for real business cycle theory (with no labor market imperfections).
- A possible solution to the puzzle: the distinction between the *extensive* margin and the *intensive* margin of labor supply.
- Extensive margin: whether or not to work; a (0,1) decision.
- Intensive margin: given that one has a job, one decides how many hours one wants to work.
- At the aggregate level, the extensive margin is much more important than the intensive margin.
- The empirical results (close to zero elasticity of supply) concern the intensive margin.

Variance decomposition for the US

- Consider following variance decomposition (Hansen, 1985)

$$Var(\tilde{h}) = Var(\widetilde{h/l}) + Var(\tilde{l}) + 2 * cov(\widetilde{h/l}, \tilde{l})$$

where $\tilde{h}$ is detrended log total hours, $\widetilde{h/l}$ is detrended log average hours and $\tilde{l}$ is detrended log employment.

- Using the US data from non-farm sector, he found

$$0.00031 = 0.000052 + 0.00017 + 0.000087$$

- Roughly 17% of the variation in total hours is due to variation in average hours worked and 55% is due to employment. Rest, 28% is due to covariance.
- Hansen argues that employment decision is (1,0) decision: either your work, or you don't. Extensive margin matters the most.

Indivisibility of labor

- Hansen (1985), and Rogerson (1988) argue that the indivisibility of labor means that there is a lot - even infinite - substitution of work over time.
- Definition of "indivisibility" (e.g. Diamond and Mirrlees (1978, 1986), Hansen (1985), Rogerson (1988): work during some time interval (say, during a week) must occur for exactly H units of time or there be no work at all.
- This indivisible environment can be contrasted with the "divisible" labor environment where workers can work any fraction of any time interval as in e.g. Lucas and Rapping (1969).

Periodic Utility

- The labor market is a frictionless spot market: workers are paid their marginal product. This wage clears the labor market, and everybody *that wants* can work.
- No involuntary unemployment, but households *can choose not to work* (and thus choose not to supply labor)
- A key property of this economy is that the elasticity of substitution of leisure in different periods if *infinite.*
- This is achieved by assuming that utility function is linear in leisure. In other words, marginal utility of leisure is constant. That is, $\mathcal{U}(C_t, L_t) = U(C_t) - v(L_t)$ is such that

$$U' > 0, U'' < 0, \ v' > 0, v'' = 0. \tag{3}$$

- But, we need some special "tricks" to justify this form of the utility function. Idea is explained in Hansen (1985).

Labor lotteries

- Assume that household either work full time, $L_t = 1$, or not at all $L_t = 0$. Assume that the utility function is given by

$$\sum_{t=0}^{T} \beta^t (U(C_t) - v(L_t)), \ v' > 0, \ v'' \geq 0 \tag{4}$$

- Assume that social planner chooses randomly a fraction of population that works and that there is a full insurance against being unemployed.
- Full insurance means that an individual gets to consume the same, whether or not (s)he works.

Labor lotteries, cont.

- The expected utility (in each period) is then

$$\begin{aligned} & E[U(C_t) - v(L_t)] \qquad (5) \\ = \;& \theta_t(U(C_t) - v(L_t = 1)) + (1 - \theta_t)[U(C_t) - v(L_t = 0)] \\ = \;& U(C_t) - \theta_t[(v(1) - v(0)] - v(0) \end{aligned}$$

where θ_t is a fraction of working population chosen by the social planner.

- (5) then simplifies to

$$U(C_t) - \psi\theta_t, \underbrace{[(v(1)]}_{\psi} > 0 \qquad (6)$$

- Replace θ_t with L_t and we get $U(C_t) - \psi L_t$.
- Thus $\mathcal{U}(C_t, L_t)$ is linear in leisure.
- We can use L_t in the production function, since this is total labor input.

Chang and Kim (2006, 2007)

- The technical device consisting of labor lotteries and full consumption insurance in Hansen (1985) is quite contrived.
- Chang and Kim (2006, 2007) study an economy with discrete employment choice at the individual level
  - work either 0 hours or H hours,
- ....but no income/consumption insurance provided by the social planner / extended family
  - Those who do not work get no labor income, and typically also consume less
- In this setting the elasticity of *aggregate* labor supply is around 1
  - The elasticity is higher than typical micro-study estimates (but still considerably lower than in Hansen's model)

Chang and Kim (2006, 2007), cont.

- Heterogeneity is important in the model
  - Individuals differ with respect to productivity
    - In each period, the most productive people tend to work, while the less productive choose not to work
    - Individual productivity may change from one period to the next (idiosyncratic productivity shocks on top of aggregate productivity shocks)
  - Individuals also differ with respect to asset holdings (productive people who have worked a lot tend to be more wealthy)
- Importantly, aggregate work and the competitive equilibrium of the economy cannot be analyzed within a representative household framework

Chang and Kim (2006, 2007), cont.

- A common feature of these models (Hansen (1985), Chang and Kim (2007) and other RBC models) is that there is no involuntary unemployment
- When there is an unfavorable (aggregate or idiosyncratic) productivity shock, some people voluntarily choose not to work.

- Chang and Kim (AER, 2007): "Heterogeneity and Aggregation: Implications for Labor Market Fluctuations"
- Chang and Kim (IER, 2006): "'From Individual to Aggregate Labor Supply: A Quantitative Analysis Based on a Heterogeneous Agent Macroeconomy"

Optimization problem with labor

- Note: We specify only the social planner's problem, since we know that competitive equilibrium is equivalent (since the welfare theorems apply).
- The social planner's problem is of the form

$$\max_{\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[U(C_t) - \psi L_t \right] \tag{7}$$

such that

$$A_t K_t^{\alpha} L_t^{1-\alpha} = C_t + \underbrace{K_{t+1} - (1-\delta) K_t}_{I_t} \tag{8}$$

$$\ln A_t = (1-\rho)\,\bar{A} + \rho A_{t-1} + \epsilon_t,\ \epsilon_t \text{ is } iid \tag{9}$$

Optimization problem with labor, cont.

- The planner's value function $V(K_t, A_t)$ satisfies the recursive Bellman equation

$$
\begin{aligned}
V(K_t, A_t) &= \max_{C_t, K_{t+1}, L_t} U(C_t) - \psi L_t + E_t\left[V(K_{t+1}, A_{t+1})\right] \\
& \quad s.t. \\
A_t K_t^{\alpha} L_t^{1-\alpha} &= C_t + K_{t+1} - (1-\delta) K_t \\
\ln A_t &= (1-\rho)\bar{A} + \rho A_{t-1} + \epsilon_t,\ \epsilon_t \text{ is } iid
\end{aligned}
$$

- Notice: the state variables are
  - the capital stock K_t (endogenous state variable)
  - total factor productivity A_t (exogenous stochastic process)

Optimization problem with labor, cont.

- Solve C_t from the aggregate resource constraint and plug into the utility function

$$
\begin{aligned}
V\left(K_t, A_t\right) &= \max_{K_{t+1}, L_t} U(A_t K_t^{\alpha} L_t^{1-\alpha} + (1-\delta)K_t - K_{t+1}) - \psi L_t \\
&\quad + \beta E_t\left[V\left(K_{t+1}, A_{t+1}\right)\right] \qquad (10) \\
&\quad st. \\
\ln A_t &= (1-\rho)\bar{A} + \rho A_{t-1} + \epsilon_t, \ \epsilon_t \text{ is } iid
\end{aligned}
$$

- First order conditions

$$
\begin{aligned}
-U'\left(C_t\right) + \beta E_t\left[V'\left(K_{t+1}, A_{t+1}\right)\right] &= 0\,(\text{wrt } K_{t+1}) \quad (11) \\
U'\left(C_t\right)(1-\alpha) A_t \left(\frac{K_t}{L_t}\right)^{\alpha-1} - \psi &= 0\,(\text{wrt } L_t) \quad (12)
\end{aligned}
$$

Optimization problem with labor, cont.

- Differentiate the Bellman equation (10) with respect to K_t:

$$V'\left(K_t, A_t\right) = \underbrace{U'\left(C_t\right)\left(\alpha A_t \left(\frac{K_t}{L_t}\right)^{\alpha-1} + 1 - \delta\right)}_{\text{direct effect}}$$
$$\underbrace{+\left\{-U\left(C_t\right) + \beta E_t\left[V'\left(K_{t+1}, A_{t+1}\right)\right]\right\}\frac{dK_{t+1}}{dK_t} + \left\{U'\left(C_t\right)\alpha A_t\left(\frac{K_t}{L_t}\right)^{\alpha} - \psi\right\}\frac{dL_t}{dK_t}}_{\text{indirect effect}}$$

- Due to FOCs (11) and (12), the indirect effect can be ignored.
- Only the direct effect remains, yielding the envelope condition:

$$V'\left(K_t, A_t\right) = U'\left(C_t\right)\left(\alpha A_t \left(\frac{K_t}{L_t}\right)^{\alpha-1} + 1 - \delta\right) \tag{13}$$

Consumption Euler equation and labor-leisure choice

- Lead the envelope condition (13) by one period and take expectations. Then plug it into the FOC (11), to obtain:

$$E_t\left[\underbrace{\frac{U'(C_t)}{\beta U'(C_{t+1})}}_{\substack{\text{marginal rate of}\\ \text{intertemporal substitution}}}\right] = \underbrace{E_t\left[A_{t+1}\alpha\left(\frac{K_{t+1}}{L_{t+1}}\right)^{\alpha-1}+1-\delta\right]}_{\substack{\text{additional production }+\\ \text{capital left over after production}}}$$

- From (12) we get

$$\underbrace{\psi}_{\text{marginal disutility from labor}} = \underbrace{(1-\alpha)A_t\left(\frac{K_t}{L_t}\right)^{\alpha}\times U'(C_t)}_{\substack{\text{marginal product of labor}\\ \times\text{ marginal utility of consumption}}}$$

- The last condition equates the cost of marginal increase in work ψ with marginal benefit.

Full RBC model: dynamic system with log utility

- Given the FOCs, the stochastic process, and the aggregate resource constraint, the full RBC model is characterized by the following equations:

$$
\begin{aligned}
1 &= \beta E_t \left[\left(\frac{C_t}{C_{t+1}} \right) \underbrace{\left(\alpha A_{t+1} \left(\frac{K_{t+1}}{L_{t+1}} \right)^{\alpha-1} + 1 - \delta \right)}_{R_{t+1}} \right] \\
\psi C_t &= \underbrace{(1-\alpha) A_t \left(\frac{K_t}{L_t} \right)^{\alpha}}_{W_t} \\
A_t K_t^{\alpha} L_t^{1-\alpha} &= C_t + \underbrace{K_{t+1} - (1-\delta) K_t}_{I_t} \\
\ln A_t &= (1-\rho) \ln \bar{A} + \rho \ln A_{t-1} + \epsilon_t,\ \epsilon_t \text{ is } iid
\end{aligned}
$$

Full RBC model, cont.

- 2 first order conditions + 1 aggregate constraint
- 3 equations and 3 endogenous variables: (C_t, L_t, K_{t+1})

- We can also easily compute additional macro variables, with the help of C_t, L_t and K_t:
- Output $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$
- Investment $I_t = Y_t - C_t$
- Real wage $W_t = (1-\alpha)\, Y_t / L_t = (1-\alpha)\, A_t \,(K_t / L_t)^{\alpha}$
- Gross real interest rate
  $R_t = \alpha Y_t / K_t + 1 - \delta = \alpha A_t \,(K_t / L_t)^{\alpha - 1} + 1 - \delta$

Steady state

The steady state values of C_t, L_t, K_t are characterized by the following equations:

$$
\begin{aligned}
1 &= \beta \underbrace{\left(\alpha \bar{A} \left(\frac{\bar{K}}{\bar{L}} \right)^{\alpha - 1} + 1 - \delta \right)}_{\bar{R}} && (14) \\
\psi \bar{C} &= \underbrace{(1-\alpha) \bar{A} \left(\frac{\bar{K}}{\bar{L}} \right)^{\alpha}}_{\bar{W}} && (15) \\
\delta \bar{K} &= \underbrace{\overbrace{\bar{A} \bar{K}^{\alpha} \bar{L}^{1-\alpha}}^{\bar{Y}} - \bar{C}}_{\bar{I}} && (16)
\end{aligned}
$$

These equations also embody an implicit characterization of the steady state values of R_t, W_t, Y_t and I_t.

Steady state calibration step by step

- Equations (14)-(16) constitute a "closed" system of 3 endogenous equations in 3 unknowns.
- The system also includes 5 parameters and exogenous variables: α, β, δ, ψ and $\bar{A}$
- Therefore, to calibrate the model, we have to impose 5 restrictions.
- We start with the normalization $\bar{A} = 1$ (restriction 1).
- We also set $\bar{r}$ to an empirical value (restriction 2). This (modified golden rule) gives us the value for β: $\bar{r} = \bar{R} - 1 = \frac{1}{\beta} - 1$
- β is commonly chosen to match (annual) net interest rate of 4%:

$$
\begin{aligned}
(1 + 0.04)^{.25} &= \frac{1}{\beta} \\
\beta &= 0.99
\end{aligned}
$$

Steady state calibration step by step, cont.

- Set $\alpha = 0.36$ to match the long-run capital income-to-GDP ratio of 36% (restriction 3):

$$\underbrace{Y}_{\text{GDP}} = \underbrace{\alpha \frac{Y}{K} K}_{\text{Capital income}} + \underbrace{(1-\alpha)\frac{Y}{L} L}_{\text{Labor income}}$$

- We set $\delta = 0.025$ (restriction 4). In the steady state

$$\begin{aligned} \bar{I} &= \delta \bar{K} \\ \frac{\bar{I}}{\bar{Y}} / \frac{\bar{K}}{\bar{Y}} &= \delta \end{aligned}$$

With empirically plausible $\bar{I}/\bar{Y}$ of 25.64%, and $\bar{K}/\bar{Y}$ of 10.2561 (2.564 for annual GDP), we get

$$\delta = \frac{0.2564}{10.2561} = 0.025$$

Steady state calibration step by step, cont.

- In order to finish the solution of the NSSS, we need to impose one more restriction.
- For example, we could normalize $\bar{L}$ to some empirically justified value.
- We could calibrate ψ directly.
- We could impose some "big ratio" condition ($\frac{\bar{K}}{\bar{Y}}$ or $\frac{\bar{C}}{\bar{Y}}$).

- We proceed with normalizing $\bar{L} = 0.3335$ (restriction 5). This reflects the belief that people work roughly $1/3$ of their non-sleeping time.

Steady state calibration step by step, cont.

- From (14) we can solve for $\bar{K}$ as:

$$\bar{K} = \left(\frac{\alpha \bar{A}}{\bar{r} + \delta} \right)^{\frac{1}{1-\alpha}} \bar{L}$$

- Resource constraint (16) then gives us the value for $\bar{C}$:

$$\bar{C} = \bar{A}\bar{K}^{\alpha}\bar{L}^{1-\alpha} - \delta\bar{K}$$

- Finally, we obtain ψ from (15):

$$\psi = (1-\alpha)\,\bar{A}\left(\frac{\bar{K}}{\bar{L}} \right)^{\alpha} \bar{C}^{-1}$$

Additional remarks about the steady state

- From (14) and (15) we can also see that $\frac{\bar{K}}{\bar{L}}$, $\bar{C}$ and $\bar{W}$ are all proportional to $\bar{A}^{\frac{1}{1-\alpha}}$, hence they are increasing in NSSS TFP:

$$\frac{\bar{K}}{\bar{L}} = \left(\frac{\alpha \bar{A}}{\bar{r} + \delta} \right)^{\frac{1}{1-\alpha}} \tag{17}$$

$$\bar{C} = \frac{(1-\alpha)}{\psi} \left(\frac{\alpha}{\bar{r} + \delta} \right)^{\frac{\alpha}{1-\alpha}} \bar{A}^{\frac{1}{1-\alpha}} \tag{18}$$

$$\bar{W} = (1-\alpha) \bar{A}^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{\bar{r} + \delta} \right)^{\frac{\alpha}{1-\alpha}} \tag{19}$$

Additional remarks about the steady state, cont.

- Steady state hours $\bar{L}$ can be expressed as:

$$
\begin{aligned}
\bar{L} &= \bar{K}\left(\frac{\bar{L}}{\bar{K}}\right) = \underbrace{\left(\frac{(1-\alpha)\left(\frac{\alpha}{\bar{r}+\delta}\right)^{\frac{\alpha}{1-\alpha}}}{\psi\left(\frac{\bar{r}+\delta}{\alpha}-\delta\right)}\bar{A}^{\frac{1}{1-\alpha}}\right)}_{\bar{K}}\underbrace{\left(\frac{\alpha\bar{A}}{\bar{r}+\delta}\right)^{-\frac{1}{1-\alpha}}}_{\left(\frac{\bar{L}}{\bar{K}}\right)} \\
&= \frac{(1-\alpha)}{\psi\left(\frac{\bar{r}+\delta}{\alpha}-\delta\right)}\left(\frac{\bar{r}+\delta}{\alpha}\right) = \frac{1}{\psi}\left[1+\frac{\alpha}{1-\alpha}\left(\frac{r}{r+\delta}\right)\right]^{-1}
\end{aligned}
$$

- Notice that $\bar{L}$ is independent from $\bar{A}$. With logarithmic utility income and substitution effects cancel each other out exactly. But households still substitute labor inter-temporally. So they do react to *temporary* movements in technology. We wanted this substitution to be high, so we assumed that marginal disutility of labor is linear.

Additional remarks about the steady state, cont.

- Another look at steady state hours worked $\bar{L}$: permanent change in real income does not affect labor supply.
- The basic intuition can be grasped by considering a simple maximization problem (see the beginning of these notes!).
- Assume that the economy is in steady state.
- An individual household chooses its consumption C and labor supply L.
- It takes as given the wage rate $\bar{W}$, and the net real interest rate $\bar{r}$.
- We also assume that the household owns the economy-wide average share of the capital stock $\bar{K}$

Household's problem in Hansen's model

- $$\max_{C,L} \ln(C) - v(L)$$
  $$\text{s.t. } C = WL + rK$$
- The first-order condition is

$$\frac{\overbrace{\bar{W}}^{\text{substitution effect}}}{\underbrace{\bar{W}L + \bar{r}\bar{K}}_{\text{income effect}}} = v'(L) \Leftrightarrow \frac{\bar{W}}{L\left(\bar{W} + \bar{r}\frac{\bar{K}}{L}\right)} = v'(L) \quad (= \psi)$$

- In the symmetric equilibrium $L = \bar{L}$. Then

$$\frac{\bar{W}}{\bar{L}\left(\bar{W} + \bar{r}\frac{\bar{K}}{\bar{L}}\right)} = v'(\bar{L}) \quad (= \psi)$$

Household's problem in Hansen's model, cont.

- Recall that both $\bar{W}$ and $\frac{\bar{K}}{\bar{L}}$ are proportional to $\bar{A}^{\frac{1}{1-\alpha}}$. In particular, $\frac{\bar{K}}{\bar{L}}$ can be expressed as follows

$$\frac{\bar{K}}{\bar{L}} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1}{\bar{r}+\delta}\right)\bar{W}$$

Hence

$$\frac{\overbrace{\bar{W}}^{\text{substitution effect}}}{\underbrace{\bar{W}}_{\text{income effect}}\quad \bar{L}\left(1+\frac{\alpha}{1-\alpha}\frac{\bar{r}}{\bar{r}+\delta}\right)} = v'\left(\bar{L}\right) \quad (=\psi)$$

- Clearly, the substitution effect and the income effect cancel out each other, and $\bar{L}$ does not depend on $\bar{W}$ (or $\bar{A}$)

$$\bar{L} = \frac{1}{\psi}\left[1+\frac{\alpha}{1-\alpha}\frac{\bar{r}}{\bar{r}+\delta}\right]^{-1}$$

Calibrating the stochastic process

- TFP constant $\bar{A}$ has been normalized to 1. One could choose it also so that NSSS output $\bar{Y} = 1$, for example.
- We are left with finding the persistence and volatility of the stochastic process ρ and σ_ϵ^2.
- We can estimate the technology process for a chosen α:

$$\begin{aligned} \ln Y_t &= \ln A_t + \alpha \ln K_t + (1-\alpha) \ln L_t \\ \ln A_t &= \ln Y_t - \alpha \ln K_t - (1-\alpha) \ln L_t \end{aligned}$$

  i.e. the Solow residual.
- Notice: The data has been HP-filtered to remove the trend
- We can estimate ρ and σ_ϵ^2 by running OLS on

$$\ln A_t = c + \rho \ln A_{t-1} + \epsilon_t$$

  and setting $\rho = \widehat{\rho}$ and $\sigma_\epsilon^2 = \frac{1}{T} \sum \epsilon_t^2$.
- Estimates from the US data $\rho = 0.74$, $\sigma_\epsilon = 0.0066$.
- Alternative persistence (to be used below as well) $\rho = 0.95$

Analyzing nonlinear DSGE models

- Derive first-order conditions (by hand)
- *Solve deterministic steady state/balanced growth path (by hand, or by computer)*
- Calibrate parameter values
- *Linearize the intra- and inter-temporal optimality conditions around deterministic steady state* (by hand, or by computer)
- *Compute respective policy and transition functions* (by computer)
- Analyze the models by means of impulse responses and moments (by computer)

Propagation of shocks in the RBC model: labor supply

- One key finding from this model is that *temporarily* high productivity (high A_t) will induce workers to work more. This endogenously propagates the technology shocks.
- People work harder to save for tomorrow. In this way they can have higher consumption also tomorrow, when productivity will be lower again (consumption smoothing).
- If instead productivity is expected to be permanently higher (very persistent positive shock to A_t, ρ close to 1), there is no need to work harder today: in any case people can enjoy high consumption in the future (due to high productivity).

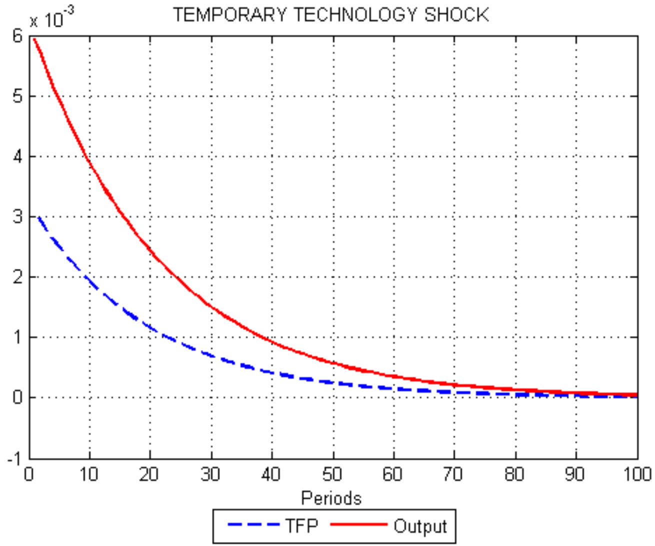
pure productivity shocks (no persistence)

- Kydland and Prescott (1982) argued that about 70 percent of the BC fluctuations can be explained by technology shocks: That's the reason for all the fuzz...
- Endogenous supply of labor will propagate these shocks into the economy (a basic idea of business cycle theory)
- This propagation is, however, weak because labor supply simply responds to current technology
- Furthermore, investment ($I_t \equiv Z_t K_t^{\alpha} L_t^{1-\alpha} - C_t$) responds to current technology.
- The problem is that fluctuations of output and hours are *persistent* (see table Business Cycle Statistics, US (non-farm) (1951Q1-2004Q4)).

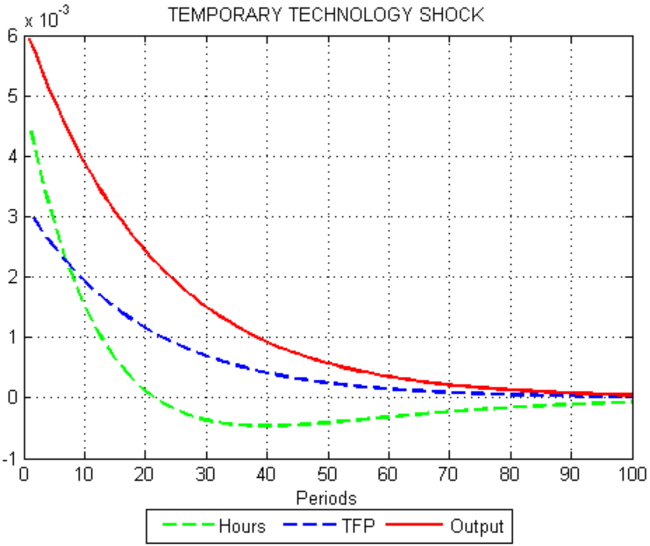
Persistent technology shocks

- Assuming that technology shocks are persistent helps in some dimensions.
- *Direct channel*: persistent fluctuations translate directly into persistent fluctuations in output (exogenous amplification and propagation of shocks).
- *Expectations channel*: A good shock today implies that tomorrow's shock will be good as well. If households (planner) are forward looking (as they are), this will induce a strong reaction of investment to technology. Why?
- The drawback is that labor supply responds less strongly to persistent technology shocks.

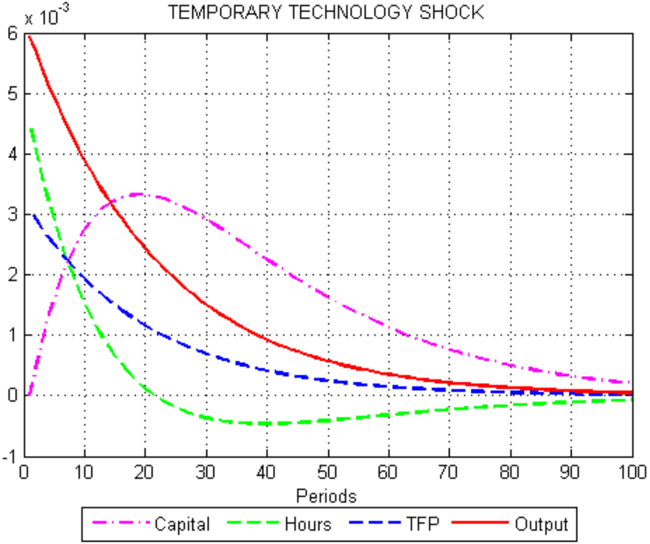
Impulse responses: output and technology, $\rho = 0.95$



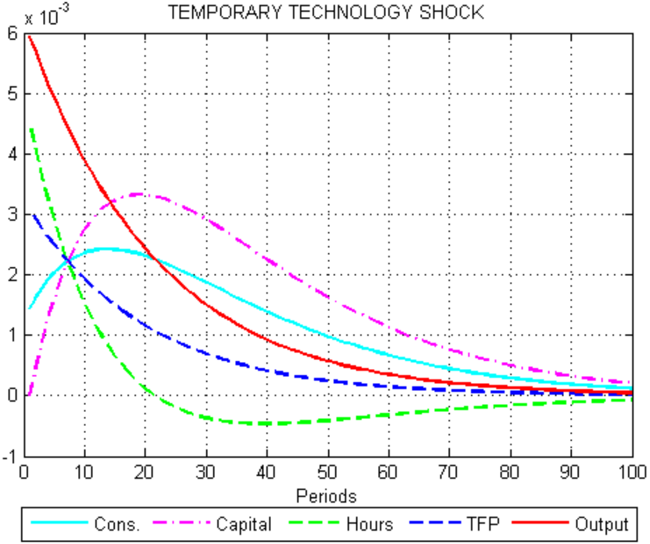
Impulse responses: hours, output and technology, $\rho = 0.95$



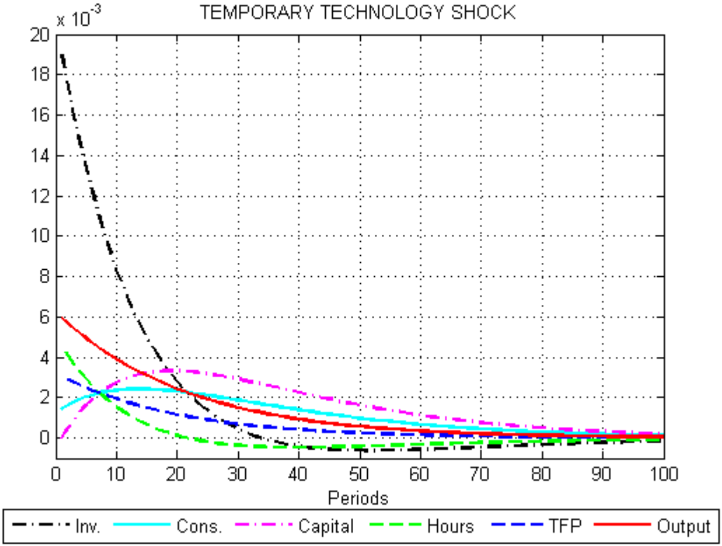
Impulse responses: capital, hours, output and technology, $\rho = 0.95$



Impulse responses: consumption, capital, hours, output and technology, $\rho = 0.95$



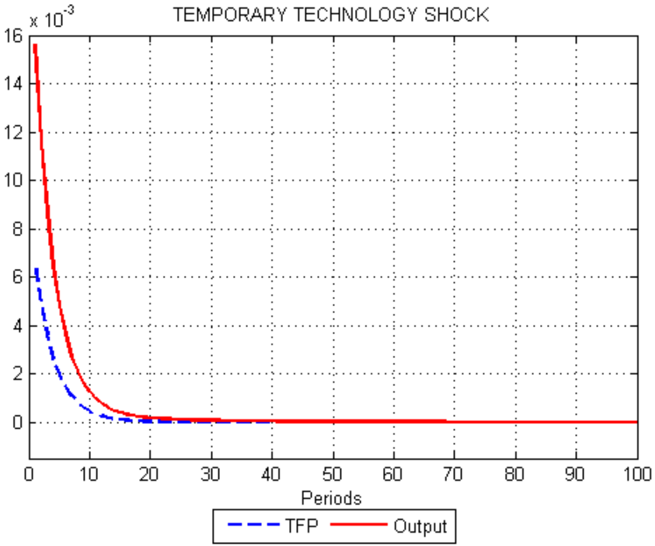
Impulse responses: investment, consumption, hours, output and technology, $\rho = 0.95$



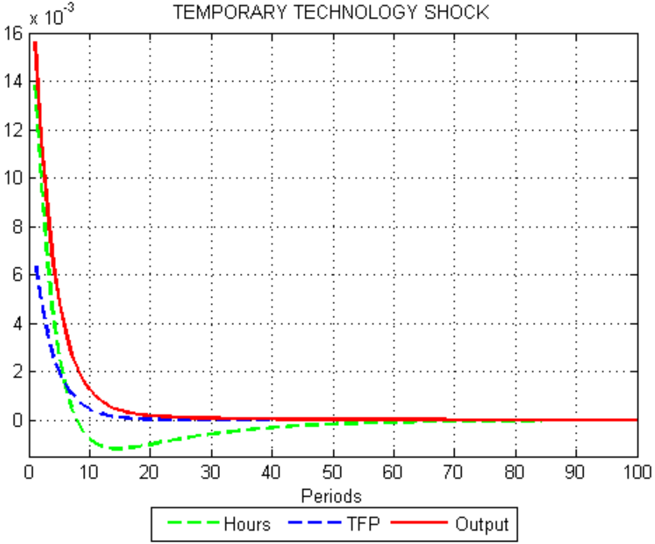
Remarks

- A positive shock to technology makes capital more productive in the future. Hence it is optimal to have more capital stock
- Labor supply responds positively, therefore output increases by more than the technology shock (intertemporal substitution effect of labor supply)
- Consumption increases less than investment. It's optimal to devote extra production to investment in order to achieve higher output in the future
- All variables return back to steady state gradually

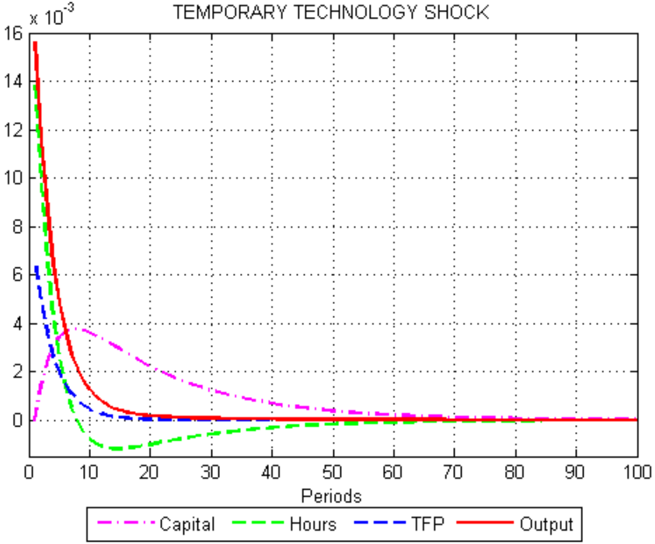
Impulse responses: output and technology, $\rho = 0.74$



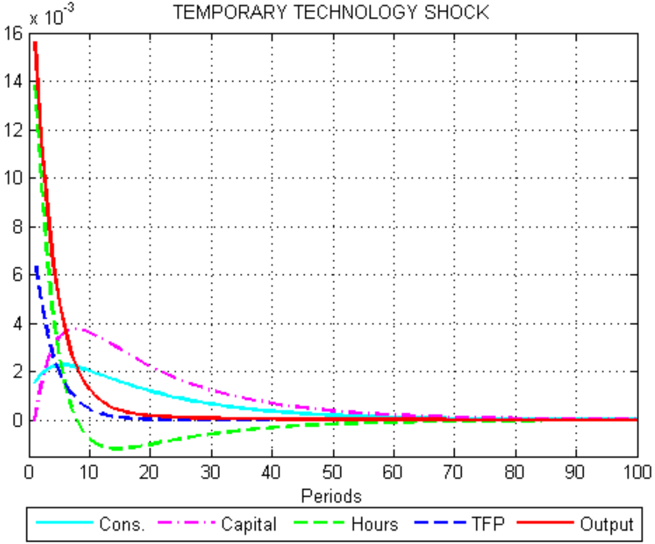
Impulse responses: hours, output and technology, $\rho = 0.74$



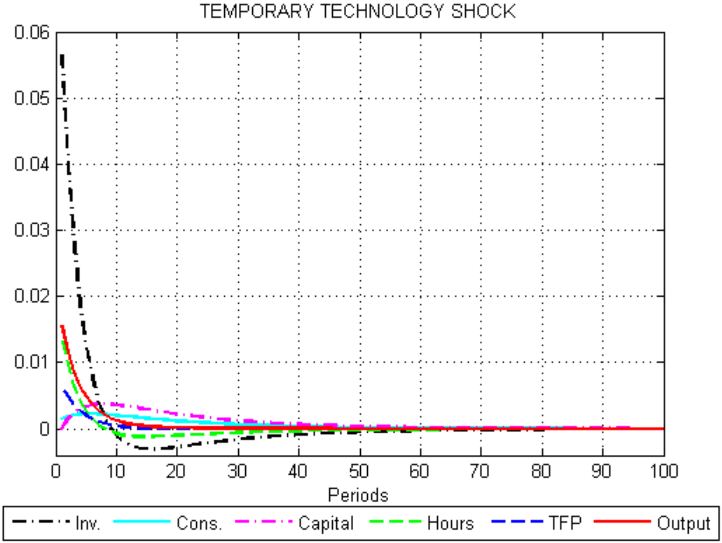
Impulse responses: capital, hours, output and technology, $\rho = 0.74$



Impulse responses: consumption, capital, hours, output and technology, $\rho = 0.74$



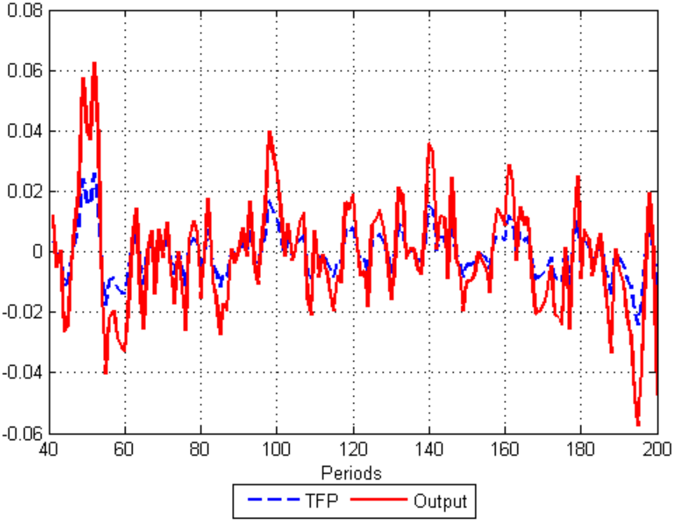
Impulse responses: investment, consumption, hours, output and technology, $\rho = 0.74$



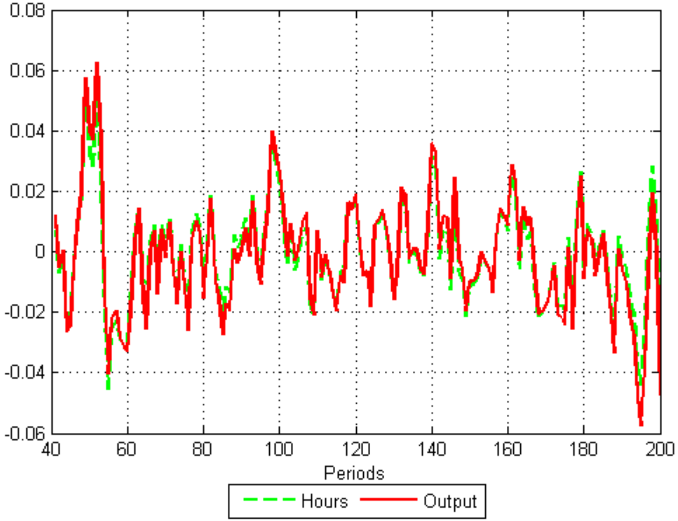
Remarks

- Initial amplification of shocks even stronger than with persistent shocks
- Very strong positive reaction of both labor supply and investment
- Since the agents expect the positive technology shock to be relatively short-lived, they want to:
  - work more today
  - save and invest more today
- Today's hard work and high savings allow for higher consumption in the future (consumption smoothing)
- However, the impact of the shock fades away more quickly than when the technology shock is more persistent
- The high level of TFP fades away quickly, so there is no reason to work hard, or to save and invest a lot

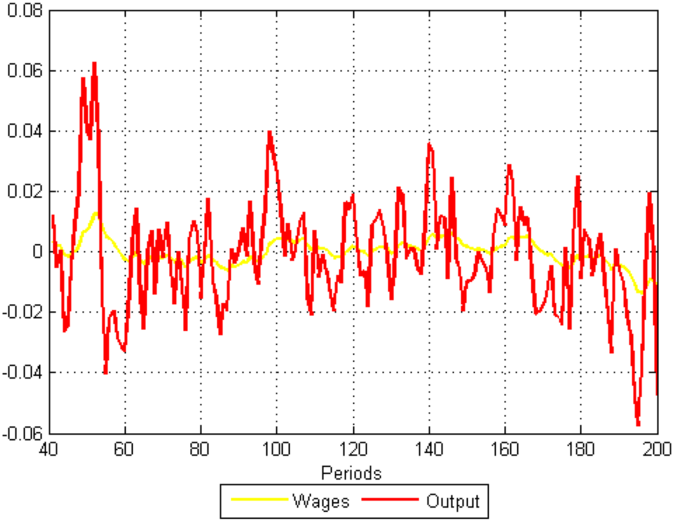
Stochastic simulations of the RBC model, $\rho = 0.74$



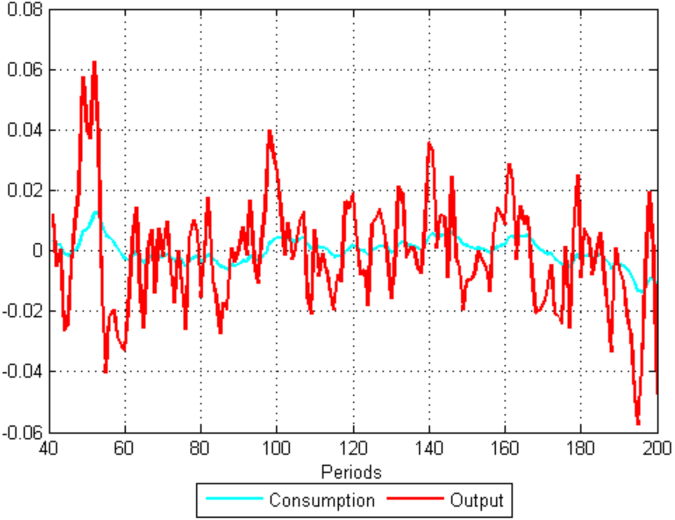
Stochastic simulations of the RBC model, $\rho = 0.74$, cont.



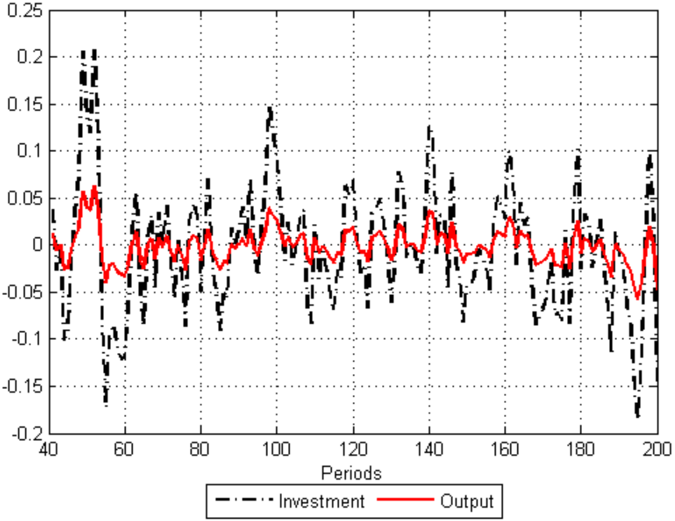
Stochastic simulations of the RBC model, $\rho = 0.74$, cont.



Stochastic simulations of the RBC model, $\rho = 0.74$, cont.



Stochastic simulations of the RBC model, $\rho = 0.74$, cont.



Business cycle statistics: The model vs. data

- Standard calibration, $\rho = 0.74$ and $\sigma_\epsilon = 0.0066$

	Standard deviation, %		Relative standard deviation compared to sd of output	
	Model	US Data	Model	US Data
y	2.35	1.81	1.0	1.0
c	0.89	1.35	0.38	0.74
i	7.93	5.30	3.37	2.98
l	1.96	1.79	0.84	0.99
w	0.89	0.68	0.38	0.38
r	0.08	0.30	0.03	0.16
a	0.98	0.98	0.42	0.54

Comments

- We now have much stronger endogenous amplification relative to the stochastic growth model.
- Volatilities of many variables, including output, investment, hours worked and wages are now stronger than in the data. Relative to output, they do a decent or good job.
- The biggest problem is that consumption is much too smooth. This is because consumption co-moves perfectly with real wages in this model (see the labor market equilibrium condition)
- Interest rates is still a disaster...

Business cycle statistics: The model vs. data

	Contemporaneous correlation with output		First order autocorrelation	
	Model	US Data	Model	US Data
y	1.0	1.0	0.75	0.84
c	0.59	0.88	0.98	0.80
i	0.96	0.80	0.70	0.87
l	0.93	0.88	0.69	0.88
w	0.59	0.12	0.98	0.66
r	0.79	-0.35	0.69	0.60
a	1.0	0.78	0.74	0.74

Comments, cont.

Correlations with output:

- Real wage clearly too procyclical in the model (roughly acyclical in the data)
- Abject failure for the real interest rate (strongly procyclical in the model, countercyclical in the data)
- These are common problems in virtually all RBC models
- The model is quite successful in replicating the correlation of employment and output

First order autocorrelations:

- Consumption and real wages are too persistent

Time-varying capacity utilization and work effort

- In RBC theory the Solow residual is often used for measuring total factor productivity and technology shocks.
- However, arguably the Solow residual reflects not only technological and organizational innovations. It also depends on cyclical (endogenous) factors such as capacity utilization (how intensively machines, instruments and buildings are used) and work effort.
- During booms the capacity utilization rate is high, and people tend to work hard.
- During a recession, capacity utilization is lower, and so is work effort (labor hoarding).
- These mechanisms increase the volatility of the Solow residual over the business cycle.

Time-varying capacity utilization and work effort, cont.

- Assume that the production function is of the form

$$Y_t = A_t \left(u_t K_t\right)^{\alpha} \left(e_t L_t\right)^{1-\alpha}$$

  where u_t is capacity utilization and e_t is work effort.

- Then the Solow residual SR_t is given by

$$\begin{aligned} \ln(SR_t) &= \ln(Y_t) - \alpha \ln\left(K_t\right) - (1-\alpha)\ln\left(L_t\right) \\ &= \ln\left(A_t\right) + \alpha \ln\left(u_t\right) + (1-\alpha)\ln\left(e_t\right) \end{aligned}$$

  and arguably a (large) portion of the variation of the Solow residual over the business cycle reflects variation in capacity utilization and work effort.

Solow residual and TFP: some empirical evidence

- There is also empirical evidence backing the view that SR is not a good measure of total factor productivity (TFP).
- If SR is computed using raw data (which has not been detrended) the difference $\ln(SR_t) - \ln(SR_{t-1})$ should measure the TFP growth rate.
- In the US data, $\ln(SR_t) - \ln(SR_{t-1}) < 0$ in more than $1/3$ of time periods.
- Thus if the growth of the Solow residual truly reflected TFP growth, the US would have experienced technological regress more than $1/3$ of the time. This is implausible.

Implications for the size of technology shocks

- Using different proxies, people have tried to measure variation in capacity utilization and work effort.
- Additional factors which may affect the Solow residual include:
  - non-constant returns to scale in production
  - imperfect competition (time-varying mark-ups)
- When these factors are taken into account, measured technology shocks (ϵ_t) become much smaller.
- Also, technological regress is a very rare event.
- More worryingly for the RBC theory, technology improvements may have little immediate effect on output, and may even decrease (nonresidential) investment on impact.
- These findings are consistent with sticky price models, but not with standard RBC models (see: Basu, Fernald and Kimball (AER, 2006): "Are technology shocks contractionary")

Capacity utilization and work effort: implications for the amplification of shocks

- However, time varying capacity utilization and work effort also provide new mechanisms through which the effects of technology shocks are amplified.
- Positive technology shock:
  - People supply more labor
  - People work harder
  - There is more investment
  - Capacity utilization rate rises
- After a negative shock the opposite happens.
- Given these amplification mechanisms, even small technology shocks can produce sizeable business cycles.